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Classical Frequentist Statistics: Counterpoint

by William M. Briggs

Since Professor Hook asked his audience to make the calculation about Briggs being right, we'll do the same. Given the many arguments and proofs he presented, the probability that Briggs, which is to say I, am right that classical statistics should be abandoned forthwith is 1. That is, it is certain I am correct. This is not the Bayesian answer, nor the frequentist answer, the false dichotomy the good Professor offers. My answer is the plain old probability answer. It is not frequentist, it is not Bayesian.

My view of probability is that favored by (to show how easy it is to provide a list of named sources) Laplace, Harold Jeffreys, John Maynard Keynes, Richard Threlkeld Cox, David Stove, ET Jaynes, a growing number of others, and, I hope, soon everybody. We call it just "probability." It's the easiest thing in the world to understand. Take all your assumptions, premises, data, and everything you consider relevant or guess might be probative and use it to evaluate the chance some proposition is true.

The best thing about probability is that it can answer all questions put to it in plain language, which the other views cannot. "What's the chance this pill will cure my ill?" is a most useful question that can be answered in probability. It cannot be answered by frequentists. They would demur and say "*If* the pill does *not* work, the chance of seeing some measure larger than any seen before is some number. And that number turns out to be less than the magic number; thus I conclude the pill works." Make sense?

It better not. It is a logical fallacy. The major premise of the frequentist argument is that the pill does not work. The minor premise is something about a probability of some weird thing that was *never seen*. The conclusion is that the pill works. You cannot get from doesn't work to works in this way. But the attempt is done all the time. It's called "null hypothesis significance testing." The number which has to be less than the magic number is called the P value (Fisher's invention). Its fallacious nature, Bernoulli's Fallacy, is why I, and others, say P values must be abandoned, but quick.

The Bayesian does a better, but still imperfect, job answering the pill question. He, like the frequentist, posits some *ad hoc* parameterized probability model, then, perhaps fascinated by the math, devotes all his energy to usually just one of those parameters. He'd answer the pill question something like, "The posterior credible interval of the parameter does not contain zero." Then he'd pack up and go home, never realizing he left the patient sitting there wondering, "What in the world is a parameter?"

The probability people might use the same base models as the other fellows, but in the end Probabilists would answer the question directly, in terms anybody can understand. Patient asks "What are the chances?" and the probabilist says, "Given all I know and assume, 10 percent." Simple, right?

Does that 10 percent mean the patient should take the pill? I have no idea, and neither does any statistician. What probability is best depends entirely on the decision that must be made with it. Some patients would consider it good enough, others would not. That commonsense answer is violated, with extreme prejudice, with hypothesis testing, which makes a single decision for all patients by declaring "The pill works." Though he'd put even that in arcane language: "The null hypothesis has been rejected." And since the null hypothesis was "the pill does not work," rejecting that means concluding the pill works.

The hypothesis test advocate is therefore committed to be judge and jury. He declares a thing is true or false, end of story. The reason for this is that in frequentist theory it is forbidden, absolutely positively disallowed in all and every circumstance, to give a probability to any hypothesis. Frequentists can only say "works" or "doesn't work." If the P is wee, he says works. Of course, even though it is *the* cardinal sin of the theory, which when committed should cast out all sinners beyond the gate where there is weeping and gnashing of teeth, the frequentist will indeed *tacitly* give probabilities to hypotheses. If the P is wee, but not very wee, we'll find him saying things like "Consider also this and that..."; that is, he'll shade his language, implying "works" is not certain, as he first insisted.

He ought to lose his frequentist license for this. But he doesn't because, he figures, since he never wrote down the math of this new judgement, it doesn't count against him. Frequentism, alas, however nice in theory, has no bearing on reality. And this is all before we consider such things like the frequentist's insistence that no probability can ever be known before an "infinite number of repetitions" arrive. Instead of infinite, frequentists sometimes use the euphemism "in the long run." Discussing the oddities of this would bring us too far afield, but it was in this context that led to Keynes quipping, "In the long run we shall all be dead."

The Bayesian, again, does a better job. But still an incomplete one. The Bayesian is too busy, perhaps, in worrying about "priors," "posteriors," and Bayes factors to realize that if he just went a little farther, he could have it all, and he could put all answers in terms of simple probability. Instead, the Bayesian stops at discussing the unobservable innards of his models, an approach which rejects all the other evidence at hand, keeping only that which can be crammed into the mathematics.

There is a strange allure about Bayes and its ability to "update" probabilities. So much so that people who start thinking along these lines forget their goal, which is to state a probability given all their evidence. I'll give you a simple example.

In probability, we calculate the chance of some proposition (like "the pill works") given all evidence assumed. Here is *all* the evidence we have for a certain situation: Given "This machine must take one of states S1, S2, or S3." Conditional on that, and *only* that, we want the probability "The machine is in state S1." The answer is 1/3, as anybody can see. Suppose we later learn "The machine is malfunctioning and cannot take state S3." Given our original *and* new information, the probability "The machine is in state S1" is now one-half. Simple!

Now you could certainly use Bayes's formula to compute this; the answers necessarily must be the same. There is nothing in the world wrong with Bayes's formula. It is one of an endless number of formulas useful in probability. For those who know it, I invite them to try their hand at using it here. It takes a lot more effort than the total evidence approach. Of course, there are other times in which Bayes's formula is indispensable. But it's not magic and doesn't provide a philosophy.

The Bayesian is also apt to declare his probabilities are subjective, mere whims. This curious insistence naturally gives many pause. Why should I trust your whims? The answer is obvious: you shouldn't. In probability, the evidence assumed can be subjective, as is the proposition around which the problem revolves. But once these are both given, the probability itself, when it can be quantified, is wholly *objective*. Which evidence is allowed and which questions are the best questions to ask are matters which are, in the end, not scientific. There is no escaping this kind of subjectivity. But answers themselves are objective.

For instance, given "$7 + x = 11$," what is the probability "$x = 5$"? Not that he would, but the subjective Bayesian could give an answer other than 0, and that answer would be totally consistent with Bayesian theory. But the probabilist is forced to say 0. The frequentist, incidentally, will quail and say there is no probability, unless he can embed this homework question in an infinite sequences of similar homework questions, but each "randomly" different, and all of which converge to some limit. There really is no excuse to use frequentism.

Lastly, we come to chance. It does not exist. But, then, neither does probability. The Bayesian will usually say by implication chance is a real force, or that probability is. The frequentist must say these things, as these are the very basis of his theory. We know this because when the frequentist's P is not wee, or the Bayesian's factor is not large, they will say the results were "due to" or caused "by" chance. This is always wrong.

The probabilist says nothing can happen because of, or "by," chance. Just as nothing happens "with" a probability. No thing "has" a probability. Probability is a state of mind, not a state in the world. Things happen because of causes, which may or may not be known, and which operate under certain conditions, which again may or may not be known. Cause is not viewed in the modern, simplistic way of only the efficient cause. Cause for the probabilist is better stated as the full explanation for a proposition. The full explanation for "x = 5" is that because of the rules of arithmetic, and given the problem itself, we know this proposition must be false.

If you ask, "What is the probability this man will have a heart attack by age 70?" the probabilist will answer, "Given what assumptions?" He cannot answer without assumptions, and neither can anybody. If you knew all there was to know about this man, the state of his heart, veins, arteries, and knew all he was going to eat and how that food would interact with his body over the course of time, and all that kind of thing, then the probability would be 0 or 1, as the case may be. But if you don't know these things, then you must be uncertain of the causes and conditions, and so you are forced to consider which is the evidence best aligned with the causes and conditions you assume are or will be involved.

In the end, the heart attack will be caused by something, or it won't be caused by anything. In neither case will that cause be chance or probability. The man himself *has no probability* of heart attack. He can only be given a probability with respect to evidence assumed. Change the evidence, change the probability. And that is all probability is: the chance given all the evidence assumed, where we are very very strict about the "all."

William M. Briggs is an independent writer, scientist, and consultant. He has taught statistics atCornell University, where he earned an M.S. in atmospheric science and a Ph.D. in statistics; matt@wmbriggs.com. He is the author of Uncertainty: The Soul of Modeling, Probability & Statistics *(Springer, 2016), yet another failed attempt to convince the world to give up "frequentism." Briggs wrote "Let Go Your Wee P!," a review of Aubrey Clayton's* Bernoulli's Fallacy: Statistical Illogic and the Crisis of Modern Science*, for our Spring 2025 issue.*